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Some Remarks on the Notion of a Free Algebraic System

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SOME REMARKS ON THE NOTION OF A FREE ALGEBRAIC SYSTEM

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The notion of a free algebraic system has its origin in group theory (free groups). For some of the simplest other types of algebraic structures the corresponding notion of a free algebraic system is an immediate generalization of that of a free group. This is the case e.g. for lattices, lattices with some of the common additional requirements (modular lattices, distributive lattices, Boolean algebras), rings. A free ring is usually called a polynomial ring over the ring of integers. For more complicated algebraic structures the definition of a free system cannot be given in such a way that the desired characteristic properties of such a system all hold. A typical example of this is a field. It is well known that for a group the two following properties are characteristic for a free group F with n generators and each of them may be chosen as definition:

i. those and only those identities of words formed of generators and their inverses hold in F, which hold in all groups with n generators;

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[3] H. Orsinger, Resultantensysteme und algebraische Relationen. Math. Nachr. (im Druck.)

ii. F is a group with n generators and every mapping of its generators on n elements of an arbitrary group G may be extended to a homomorphic mapping of F into G.

Both i and ii may be easily generalized to arbitrary algebraic structures and proved to be equivalent, but in both cases a system F which satisfies the requirements need not exist. Furthermore both concepts of a "generalized word" and of a "homomorphism" may depend on the choice of the basic algebraic operations. This may already be made clear in group theory, where it makes a difference for a "word" whether one takes only the operation of multiplication or the two operations of multiplication and of forming of the inverse as basic operations.

The connection between the existence of free systems and the form of the axioms which determine the algebraic structure, can be investigated. Some results may be found in my paper "Some theorems on free algebras and direct products of algebras", Simon Stevin **29** (1951), 51—59. Here the axiom systems which are considered are simple enough to avoid the above-mentioned difficulties with the choice of the basic operations. As soon as existential statements enter into the axioms, this difficulty arises and it seems doubtful whether a reasonable definition of free system is possible at all. In some cases the existential statements may be eliminated by the introduction of new basic operations.

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